BSGTBG and Math

The probability of any specific person getting two cylons is, indeed, as follows.
(Assuming, for the moment, that Gaius Baltar is not in the game)

$$P_{\text{player gets two cylons}} = \frac{2}{12} \cdot \frac{1}{11} = \frac{2}{132} = \frac{1}{66}$$

However, that's not what the question is. The question is what the probability of *someone* getting two cylons in any given game. There are six possible distributions of cards in which *someone* has two cylons – one for each player. (The fact that the cards can be in either order is included in the probability above).

So the probability of *someone* getting two cylons is:

$$P_{\text{any player gets two cylons}} = 6 \cdot P_{\text{player gets two cylons}} = 6 \cdot \frac{1}{66} = \frac{6}{66} = \frac{1}{11} \approx 0.091$$

So, on average, one in thirteen games, someone will get both cylons. Which is more often than, for instance, Starbuck taking out a Heavy Raider in two tries.

I had to convince myself I got the math right (it's been too long since I took statistics), so here's a table that helped me get it straight in my head. C = cylon, N = not cylon.

Player 1	Player 2	Player 3	Player 4	Player 5	Player 6
CC	NN	NN	NN	NN	NN
NN	CC	NN	NN	NN	NN
NN	NN	CC	NN	NN	NN

And so on. Each row is a distinct distribution of cards, and you can calculate that the probability for each of the rows is 1 in 78, so to get the total probability for any of those situations occurring, you simply add them. While it makes sense, I also had to try one to make sure the other rows ended up canceling such that they are the same probability. Example for row two:

$$\frac{10}{12} \cdot \frac{9}{11} \cdot \frac{2}{10} \cdot \frac{1}{9} \cdot \frac{8}{8} \cdot \frac{7}{7} \cdots = \frac{\cancel{10}}{12} \cdot \frac{\cancel{9}}{11} \cdot \frac{2}{\cancel{10}} \cdot \frac{1}{\cancel{9}} \cdot 1 = \frac{2 \cdot 1}{12 \cdot 11} = \frac{1}{78}$$

Now, when Gaius Baltar gets involved, things obviously change a bit. Now, the probability of any *specific* non-Gaius player will get two cylons is slightly different, because there are 13 cards:

$$P_{\text{non-Gaius player gets two cylons}} = \frac{2}{13} \cdot \frac{1}{12} = \frac{2}{156} = \frac{1}{78}$$

And so the probability of *any* non-Gaius getting two cylons, since there are five of those players, is

$$P_{\text{any non-Gaius player gets two cylons}} \frac{1}{78} \cdot 5 = \frac{5}{78} \approx 0.064$$

Gaius gets more complicated, because the possibility of him getting any given arrangement of cylon cards is the same – 1 in 78 – but he has three possible arrangements of cards that give him two cylons (CCN, CNC, NCC). So his total chance of getting two cylons is 3 in 78, so the total chance of anyone in the came getting two cylons when Gaius is involved is:

$$P_{any\,non-Gaius\,player\,gets\,two\,cylons}+P_{Gaius\,gets\,two\,cylons}=\frac{5}{78}+\frac{3}{78}=\frac{8}{78}\approx 0.103\approx\frac{1}{10}$$

So even though there are more non-cylons in the mix, because Gaius is three times as likely to get two cylons, the overall chance of one player getting two cylons is greater than when Gaius isn't involved. Note that a non-Gaius player only has one combination that gives them two cylons (namely, CC), otherwise they would be multiplied like Gaius. Again, the different order of the cylon cards is built into the original probability.

An interesting sidenote is that if Gaius is in the game, he has 3 of the 8 possible combinations of two cylons, so over a third of the time that one player does get two cylons, it will be Gaius. Which means that the situation we were in last night – a non-Gaius player getting two cylons in a game featuring Gaius – was relatively rare as far as double-cylon events goes.

Now, one way to verify that I did the math right is to separately calculate the chance of every other possible situation, add that to the chance of a double-cylon deal, and add them to see if they come out to 1. Fortunately, there's only one other possibility (again, ignoring Gaius for the time being): the cylons are split up between two players. This is a bit more complicated, but not too much. I'll start as I did before: assuming the first two players dealt each get one cylon. For those four cards, we can calculate the number of combinations that the four cards can form:

$$nCr=\frac{4!}{2!2!}=3\cdot 2\cdot 1=6$$

The denominator is to account for the fact that there for each pair (NN and CC), swapping them creates the same combination – so you have to cut the possibilities in half for each pair. Since the number of combinations is small here, we can simply list them all: NN-CC, NC-NC, NC-CN, CN-NC, CN-CN, CC-NN. You can see here that two of our combinations are double-cylon situations, so we need to drop those, so we're left with four arrangements – NC-NC, NC-CN, CN-NC, CN-CN. We can calculate the probability of each of these, but if we do just the first:

$$P_{NC-NC}=\frac{10}{12}\cdot\frac{2}{11}\cdot\frac{9}{10}\cdot\frac{1}{9}=\frac{\cancel{10}}{12}\cdot\frac{2}{11}\cdot\frac{\cancel{9}}{\cancel{10}}\cdot\frac{1}{\cancel{9}}=\frac{2\cdot 1}{12\cdot 11}=\frac{2}{132}=\frac{1}{66}$$

We can see that there will always be 10, 9, 2, 1 on the top, and 12, 11, 10, 9 on the bottom, and they will all be the same. Which means that the probability of any specific two players getting two cylons between them is the total probability of the four permutations of cards, which is

$$P_{two\,players\,getting\,cylons}=4\cdot\frac{1}{66}=\frac{4}{66}=\frac{2}{33}$$

So now we just need to calculate the number of arrangements of two players we can make. Again, order here doesn't matter, so we have to do a little fancy math:

$$nCr=\frac{6!}{4!2!}=\frac{6\cdot 5}{2}=3\cdot 5=15$$

So, the probability of the cylons being split up, then, is:

$$P_{\text{cylons split up}}=15\,P_{\text{two players getting cylons}}=15\cdot\frac{2}{33}=\frac{30}{33}=\frac{10}{11}$$

Which very nicely meshes with our previous result of 1 in 11 games having a double-cylon deal. I could do the same math for a game with Gaius, but I have real work to do, so I'll leave that one as an exercise to the reader. Since my first math works out, I'm pretty confident in the Gaius math as well.